The Astrophysical Journal Letters, 747:L34 (5pp), 2012 March 10 doi:10.1088/2041-8205/747/2/L34

RANDOM BALLISTIC INTERPRETATION OF NONLINEAR GUIDING CENTER THEORY

D. Ruffolo[1,2], T. Pianpanit[1], W. H. Matthaeus[3], and P. Chuychai[2,4]
[1] Department of Physics, Faculty of Science, Mahidol University, Bangkok 10400, Thailand; scdjr@mahidol.ac.th, th_ee@hotmail.com
[2] Thailand Center of Excellence in Physics, CHE, Ministry of Education, Bangkok 10400, Thailand
[3] Bartol Research Institute and Department of Physics and Astronomy, University of Delaware, Newark, DE 19716, USA; yswhm@bartol.udel.edu
[4] School of Science, Mae Fah Luang University, Chiang Rai 57100, Thailand; p.chuychai@sci.mfu.ac.th
Received 2012 January 12; accepted 2012 February 8; published 2012 February 22

ABSTRACT

Nonlinear guiding center (NLGC) theory has been used to explain the asymptotic perpendicular diffusion coefficient $\kappa_\perp$ of energetic charged particles in a turbulent magnetic field, which can be applied to better understand cosmic ray transport. Here we re-derive NLGC, replacing the assumption of diffusive decorrelation with random ballistic decorrelation (RBD), which yields an explicit formula for $\kappa_\perp$. We note that scattering processes can cause a reversal of the guiding center motion along the field line, i.e., "backtracking," leading to partial cancellation of contributions to $\kappa_\perp$, especially for low-wavenumber components of the magnetic turbulence. We therefore include a heuristic backtracking correction (BC) that can be used in combination with RBD. In comparison with computer simulation results for various cases, NLGC with RBD and BC provides a substantially improved characterization of the perpendicular diffusion coefficient for a fluctuation amplitude less than or equal to the large-scale magnetic field.

Key words: diffusion – magnetic fields – turbulence

1. INTRODUCTION

While charged particles subject to a magnetic field in a tenuous plasma will mainly gyrate along that field, magnetic turbulence can cause particles to also spread in the directions perpendicular to the large-scale field. Such perpendicular transport involves an interesting interplay between the transport along field lines, the random walk of magnetic field lines perpendicular to the large-scale field direction, and true cross-field transport in which the particle guiding center eventually separates from its original field line.

The classic FLRW theory (Jokipii 1966), in which particles follow magnetic field lines with a fixed pitch angle, directly related the perpendicular diffusion coefficient $\kappa_\perp$ to the field line diffusion coefficient D. Meanwhile another viewpoint in terms of scattering led to a relation between $\kappa_\perp$ and the parallel diffusion coefficient $\kappa_\parallel$ (Axford 1965; Gleeson 1969). Nonlinear guiding center (NLGC) theory (Matthaeus et al. 2003) successfully accounted for both factors, allowing the guiding center motion to decorrelate due to both parallel (pitch-angle) scattering and the random walk of the guiding magnetic field line, for transverse magnetic fluctuations with a general power spectrum. This theory has provided a much closer match to observations (Bieber et al. 2004) and computer simulation results for $\kappa_\perp$ (see also Minnie et al. 2007; Ruffolo et al. 2008), and its framework has attracted theoretical interest and inspired numerous related theories (e.g., Shalchi et al. 2004, 2006; le Roux & Webb 2007; Qin 2007; Shalchi 2010).

The original NLGC theory (Matthaeus et al. 2003) used the Taylor–Green–Kubo (TGK) formula (Taylor 1922; Green 1951; Kubo 1957)

$$\kappa_{xx} \equiv \lim_{t\to\infty} \frac{\langle \Delta x^2 \rangle}{2t} = \int_0^\infty \langle \tilde{v}_x(0)\tilde{v}_x(t)\rangle dt \tag{1}$$

for the asymptotic particle diffusion coefficient κ_{xx} along a coordinate x perpendicular to the large-scale magnetic field direction z, based on the guiding center velocity $\tilde{\mathbf{v}}$. That work used

$$\langle \tilde{v}_x(0)\tilde{v}_x(t)\rangle \approx \frac{a^2}{B_0^2}\langle v_z(0)v_z(t)\rangle\langle b_x(0,0)b_x[\mathbf{x}(t),t]\rangle, \tag{2}$$

for the displacement $\mathbf{x}(t)$ of the particle guiding center trajectory in a large-scale magnetic field $B_0\hat{\mathbf{z}}$. The authors set $a^2 = 1/3$, a factor which effectively accounts for the replacement of $\tilde{v}_z$ with the particle velocity v_z in the correlations. Then the Lagrangian correlation $\langle b_x(0,0)b_x[\mathbf{x}(t),t]\rangle$ was evaluated in terms of the Eulerian correlation function and power spectrum by using Corrsin's independence hypothesis (Corrsin 1959) and setting the displacement distribution to that for asymptotic diffusion (Salu & Montgomery 1977), leading to an implicit formula for $\kappa_\perp$ in terms of input values of κ_{zz} and the power spectrum of magnetic fluctuations. A related approach was previously used to derive a field line diffusion coefficient (Matthaeus et al. 1995) that is reasonably close to values from direct computer simulations (Gray et al. 1996; Ghilea et al. 2011).

In the present work, we consider an alternate interpretation of NLGC that replaces the diffusive distribution of guiding center trajectories with a random ballistic distribution, for the purpose of calculating the Lagrangian magnetic correlation function $\langle b_x(0,0)b_x[\mathbf{x}(t),t]\rangle$. This approach was recently introduced for calculating the field line diffusion coefficient and led to some substantial improvements in the match with direct simulation results (Ghilea et al. 2011). It is analogous to concepts in random walk theory in which the mean free path is determined by the extent of ballistic motion between scattering events. In this context, note that $\tilde{v}_x$ decorrelates over the decorrelation scale of v_z or b_x, whichever is shorter. This implies that the decorrelation of $\tilde{v}_x$ in the TGK integral (which determines κ_{xx}) takes place over a distance scale for which the parallel motion is approximately constant and the field lines are approximately straight, so the guiding center motion can be treated as ballistic in random directions determined by the distribution of magnetic field directions (Figure 1). (As illustrated in the figure, at longer times the guiding center velocity will change, the particle will reverse its direction along $\mathbf{B}$, and the particle will depart from its original guiding field line.) We demonstrate that this

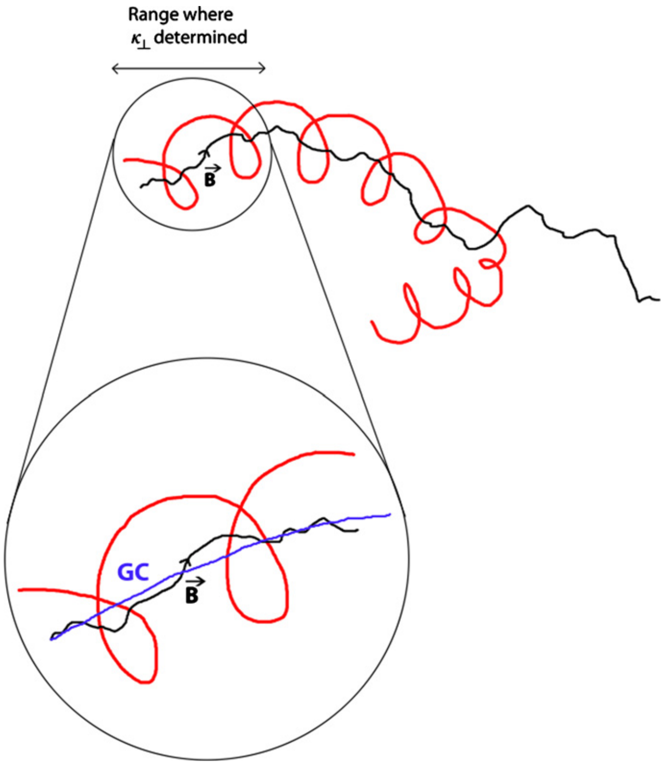

Figure 1. Illustration of the random ballistic interpretation of nonlinear guiding center (NLGC) theory. The diffusion coefficient $\kappa_\perp$ of energetic charged particle motion (red line) perpendicular to the large-scale magnetic field is related to the decorrelation (i.e., change in direction) of a perpendicular component of the guiding center velocity (GC, blue line), which roughly follows a local magnetic field line (black line). Over the relevant distance scale, the guiding center motion can be approximated as ballistic (i.e., with constant velocity) along random directions distributed like the magnetic field directions. Such random ballistic decorrelation (RBD) is determined using the framework of NLGC theory, including the effects of the field line random walk and the parallel scattering of particle trajectories.

approach, together with a backtracking correction (BC), leads to a substantial improvement in the match with direct computer simulations of the perpendicular diffusion of energetic charged particles.

2. RANDOM BALLISTIC DECORRELATION

We consider the application of Corrsin's independence hypothesis (described below) assuming a Gaussian distribution of displacements, where diffusive decorrelation (DD) or random ballistic decorrelation (RBD) is used to describe the variance σ_i^2 along each direction. DD considers that the asymptotic diffusion also governs the displacement distribution at early times during the decorrelation process, so $\sigma_i^2 = 2\kappa_{ii}t$, while RBD assumes the decorrelation is determined by ballistic motion of guiding centers at early times in random directions, at guiding center velocity $\tilde{\mathbf{v}}$, depending on the fluctuating magnetic field, with $\sigma_i^2 = \langle \tilde{v}_i^2 \rangle t^2$.

Let us assume axisymmetry, define the fluctuation amplitude b so that $b^2 = \langle b_x^2 + b_y^2 \rangle = 2\langle b_x^2 \rangle$, and define v_s as the particle velocity along the local magnetic field. As a special case of Equation (2) for $t = 0$, we use

$$\langle \tilde{v}_x^2 \rangle = \langle \tilde{v}_y^2 \rangle \approx \frac{a^2}{B_0^2} \langle v_z^2 \rangle \langle b_x^2 \rangle = \frac{a^2 v^2}{6} \frac{b^2}{B_0^2}, \tag{3}$$

where we use $\langle v_z^2 \rangle = v^2/3$ for an isotropic distribution of particle velocities. We also use $\langle \tilde{v}^2 \rangle = v^2/3$ to obtain

$$\langle \tilde{v}_z^2 \rangle = \frac{v^2}{3} \left(1 - a^2 \frac{b^2}{B_0^2} \right). \tag{4}$$

Note that for $b/B_0 > a^{-1} = \sqrt{3}$, Equation (4) gives a nonsensical negative value for $\langle \tilde{v}_z^2 \rangle$. Thus, we will consider this RBD approach to be limited to $b/B_0 \leqslant \sqrt{3}$. Note that the NLGC framework in general is also limited to magnetic fluctuation amplitudes that are not too great, in the sense that NLGC assumes transverse fluctuations, and if $b \gg B_0$ one would not expect the (weak) mean magnetic field to force the fluctuations to be strongly transverse.

As in the original derivation of NLGC, we use Equations (1) and (2), with $\langle v_z(0) v_z(t) \rangle = (v^2/3) e^{-t/\tau}$ for a pitch-angle scattering time τ, to obtain

$$\kappa_{xx} = \frac{a^2 v^2}{3B_0^2} \int_0^\infty e^{-t/\tau} \langle b_x(0,0) b_x[\mathbf{x}(t), t] \rangle dt. \tag{5}$$

We then make use of Corrsin's independence hypothesis to relate the Lagrangian correlation $\langle b_x(0,0) b_x[\mathbf{x}(t), t] \rangle$ to the Eulerian correlation function R_{xx} and the probability of displacement $\mathbf{x}$ at time t, so that

$$\kappa_{xx} = \frac{a^2 v^2}{3B_0^2} \int_0^\infty e^{-t/\tau} \int R_{xx}(\mathbf{x}, t) P(\mathbf{x}|t) d\mathbf{x}\, dt. \tag{6}$$

Following Matthaeus et al. (2003), we use the Fourier transform of the correlation function $R_{xx}(\mathbf{x}, t)$ as the power spectrum $S_{xx}(\mathbf{k}, t) = S_{xx}(\mathbf{k}) e^{-\gamma(\mathbf{k})t}$ and assume independent guiding center displacement probability distributions along each coordinate to obtain

$$\kappa_{xx} = \frac{a^2 v^2}{3B_0^2} \int_0^\infty e^{-t/\tau} \int S_{xx}(\mathbf{k}) e^{-\gamma(\mathbf{k})t} \left(\int_{-\infty}^\infty e^{-ik_x x} P(x|t) dx \right) \times \left(\int_{-\infty}^\infty e^{-ik_y y} P(y|t) dy \right) \left(\int_{-\infty}^\infty e^{-ik_z z} P(z|t) dz \right) d\mathbf{k}\, dt. \tag{7}$$

For a Gaussian displacement distribution $P(x|t)$, we have (Ghilea et al. 2011)

$$\int_{-\infty}^\infty e^{-ik_x x} P(x|t) dx = \exp\left(-\frac{1}{2} k_x^2 \sigma_x^2 \right) \tag{8}$$

and analogous formulas for y and z. For RBD we use $\sigma_i^2 = \langle \tilde{v}_i^2 \rangle t^2$, and substituting Equation (8) into Equation (7) yields

$$\kappa_{xx} = \frac{a^2 v^2}{3B_0^2} \int S_{xx}(\mathbf{k}) T(\mathbf{k})\, d\mathbf{k}, \tag{9}$$

where the mean free time $T(\mathbf{k})$ is given by

$$T(\mathbf{k}) = \int_0^\infty \exp\left[-\frac{t}{\tau} - \gamma(\mathbf{k})t - \frac{1}{2} \sum_i k_i^2 \langle \tilde{v}_i^2 \rangle t^2 \right] dt. \tag{10}$$

Performing the t-integration and using $1/\tau = v/\lambda_\parallel = v^2/(3\kappa_{zz})$, we obtain

$$T(\mathbf{k}) = \sqrt{\frac{\pi}{2}} \frac{e^{\alpha^2} \operatorname{erfc}(\alpha)}{\sqrt{\sum_i k_i^2 \langle \tilde{v}_i^2 \rangle}} \tag{11}$$

and

$$\kappa_{xx} = \frac{a^2 v^2}{3B_0^2}\sqrt{\frac{\pi}{2}}\int \frac{S_{xx}(\mathbf{k})}{\sqrt{\sum_i k_i^2\langle\tilde{v}_i^2\rangle}} e^{\alpha^2}\,\mathrm{erfc}(\alpha) d\mathbf{k} \qquad \text{(RBD)}, \tag{12}$$

where

$$\alpha \equiv \frac{v^2/(3\kappa_{zz}) + \gamma(\mathbf{k})}{\sqrt{2\sum_i k_i^2\langle\tilde{v}_i^2\rangle}} \tag{13}$$

and the expressions for $\langle\tilde{v}_i^2\rangle$ are given by Equations (3) and (4).

Note that the original DD interpretation of Matthaeus et al. (2003) used the formula for asymptotic diffusion with κ_{xx} in the displacement distribution, yielding an implicit equation for κ_{xx}. In contrast, the RBD theory uses a predetermined random ballistic formula for the displacement distribution and yields an explicit formula for κ_{xx}, as in analogous theories for the field line diffusion coefficient (Ghilea et al. 2011). For numerical evaluation, NLGC-type theories based on DD typically require an iterative solution, whereas NLGC/RBD can be evaluated without iteration.

3. BACKTRACKING CORRECTION

Previous simulations have shown that the perpendicular transport of energetic charged particles is characterized by ballistic (free-streaming) guiding center motion at short times, followed by subdiffusion (Qin et al. 2002a) and later, if the fluctuations have sufficient transverse complexity, by asymptotic diffusion (Qin et al. 2002b). This subdiffusion is due to a parallel (pitch-angle) scattering process that causes a particle to reverse its motion along the local field line and partially retrace its steps. Such "backtracking" leads to a negative v_x-correlation function over a certain time range, hence the reduction in the running perpendicular diffusion coefficient. In some cases this leads to subdiffusion (see Qin et al. 2002b and Section 4 of Ruffolo et al. 2008).

Backtracking was inherent in the original NLGC/DD theory (Matthaeus et al. 2003). The use of diffusive displacements means that the displacements for which the correlation function is sampled can undergo a random walk, including backtracking. It was assumed that backtracking did not completely cancel out the perpendicular guiding center excursions due to other physical effects. This is not the case for the RBD calculation, which is based on ballistic guiding center trajectories.

For RBD theory, we note that Equations (9) and (10) assign a mean free time $T(\mathbf{k})$ to individual $\mathbf{k}$-components of the turbulence, which are averaged with weighting according to the power spectrum, to determine κ_{xx}. Conceptually this relates to the v_z–b_x independence hypothesis of Matthaeus et al. (2003). For magnetostatic fluctuations with $\gamma = 0$, Equation (11) gives $T \approx \tau$ for low k and T decreases for higher k. Thus, for modes of low k, the mean free time is determined by the parallel scattering, whereas for higher k it is determined by the field line random walk.

This random ballistic calculation of the mean free time does not account for backtracking. Consider low k, for which the decorrelation in Equation (10) is dominated by the scattering term (first term in the exponential) while $\mathbf{b}$ is nearly constant. Then the perpendicular displacement associated with $T(\mathbf{k})$ will be largely canceled out by subsequent backtracking. A similar effect leads to subdiffusion in simulation results (i.e., running κ_{xx} decreases with increasing t) for fluctuations with insufficient transverse complexity (Qin et al. 2002a), whereas NLGC yields a much larger asymptotic value of κ_{xx} (see Run 12 of Ruffolo et al. 2008).

Therefore, we introduce a heuristic BC for RBD that reduces the influence of such low-k modes by reducing $T(\mathbf{k})$ and therefore their contribution to the overall κ_{xx}. We multiply $T(\mathbf{k})$ by $e^{-\alpha^2}$, which simplifies Equation (11) to yield

$$T(\mathbf{k}) = \sqrt{\frac{\pi}{2}}\frac{\mathrm{erfc}(\alpha)}{\sqrt{\sum_i k_i^2\langle\tilde{v}_i^2\rangle}} \tag{14}$$

and

$$\kappa_{xx} = \frac{a^2 v^2}{3B_0^2}\sqrt{\frac{\pi}{2}}\int \frac{S_{xx}(\mathbf{k})}{\sqrt{\sum_i k_i^2\langle\tilde{v}_i^2\rangle}} \times \mathrm{erfc}\left[\frac{v^2/(3\kappa_{zz}) + \gamma(\mathbf{k})}{\sqrt{2\sum_i k_i^2\langle\tilde{v}_i^2\rangle}}\right] d\mathbf{k} \qquad \text{(RBD/BC)}. \tag{15}$$

This BC is related to the terms that are linear and quadratic in t, in the exponential of Equation (10). Here, $e^{-\alpha^2}$ serves as a simple "switch" that is close to 0 when k is sufficiently low that the linear term dominates, suggesting a strong effect of backtracking, while it approaches 1 for higher k. Note also that for a given $\mathbf{k}$, there is a time t when the linear and quadratic terms are equal, i.e., the field line random walk becomes important. At that time we have $t/\tau \sim \alpha^2$, and substitution into the parallel velocity correlation term $e^{-t/\tau}$ suggests the use of $e^{-\alpha^2}$ to account for backtracking effects.

4. NUMERICAL EVALUATION OF ANALYTIC THEORIES USING 2D+slab TURBULENCE

To numerically evaluate analytic theories for comparison with computer simulation results, we need to specify the power spectrum. We employ a two-component 2D+slab model of transverse magnetic fluctuations in which the power spectrum is a sum of a two-dimensional (2D) power spectrum, depending on k_x and k_y, and a slab power spectrum depending on k_z. The latter represents parallel Alfvénic fluctuations and the former idealizes the quasi-2D structures, including "flux tubes," that can develop from interactions of such waves (Shebalin et al. 1983; see also Borovsky 2008; Seripienlert et al. 2010; and references therein). The two-component model was motivated by observations of interplanetary magnetic fluctuations, indicating quasi-slab and quasi-2D components (Matthaeus et al. 1990; Weygand et al. 2009), which can be modeled using a ratio of slab:2D fluctuation energies of approximately 20:80 (Bieber et al. 1994, 1996). This model has provided a useful description of the parallel transport of particles in the inner heliosphere (Bieber et al. 1994), and was used by most studies that implemented and/or tested NLGC theory.

For the special case of 2D+slab fluctuations, Equations (12) and (15) and their DD equivalent split into two terms using S_{xx}^{slab} and S_{xx}^{2D}. However, Shalchi (2006) has proposed that the direct contribution of slab fluctuations to the perpendicular transport should be subdiffusive, and that the S_{xx}^{slab} term should not be included in the equation of $\kappa_\perp$. (Note that slab fluctuations can still play a role as a key determinant of $\lambda_\parallel$, which enters into the 2D contribution.) We refer to this proposal as the Shalchi slab hypothesis. We employ this in the present work, and a detailed evaluation of its accuracy will be presented in a future publication (D. Ruffolo et al., in preparation).

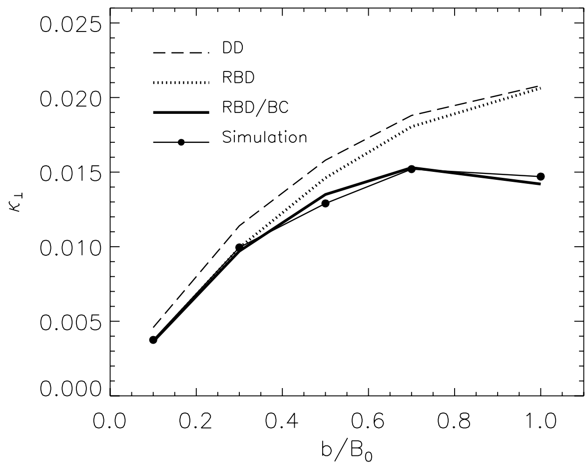

Figure 2. Asymptotic perpendicular diffusion coefficient $\kappa_\perp$ of 100 MeV protons in 2D+slab turbulence with a slab fraction $f_s = 0.2$ as a function of the magnetic fluctuation amplitude b/B_0. Using the NLGC framework, random ballistic decorrelation with backtracking correction (RBD/BC, thick line) provides a closer match with computer simulation results (solid circles) than the original DD theory (long-dashed line) and uncorrected RBD (short-dashed line). In the present work we also employ the Shalchi slab hypothesis (Shalchi 2006).

Therefore, when using the 2D+slab model of magnetic turbulence, in Equation (12) or Equation (15) we include only the 2D portion of the power spectrum, which is concentrated at $k_z = 0$. We also assume the fluctuations to be magnetostatic, with $\gamma = 0$, and axisymmetric. For RBD without the BC, we have

$$\kappa_{xx} = \frac{a^2 v^2}{3B_0^2}\sqrt{\frac{\pi}{2}}\int_{-\infty}^{\infty}\int_{-\infty}^{\infty} \times \frac{S_{xx}^{2D}(k_x,k_y)}{k_\perp\sqrt{\langle\tilde{v}_x^2\rangle}}\, e^{\alpha^2}\,\mathrm{erfc}(\alpha)\,dk_x\,dk_y \qquad \text{(RBD)}, \qquad (16)$$

and with the BC we have

$$\kappa_{xx} = \frac{a^2 v^2}{3B_0^2}\sqrt{\frac{\pi}{2}}\int_{-\infty}^{\infty}\int_{-\infty}^{\infty} \times \frac{S_{xx}^{2D}(k_x,k_y)}{k_\perp\sqrt{\langle\tilde{v}_x^2\rangle}}\,\mathrm{erfc}(\alpha)\,dk_x\,dk_y \qquad \text{(RBD/BC)}, \qquad (17)$$

where

$$\alpha = \frac{v^2}{3\kappa_{zz}k_\perp\sqrt{2\langle\tilde{v}_x^2\rangle}} \qquad (18)$$

and $k_\perp^2 = k_x^2 + k_y^2$.

For comparison, we also consider the original DD theory, and for our model assumptions we obtain

$$\kappa_{xx} = \frac{a^2 v^2}{3B_0^2}\int_{-\infty}^{\infty}\int_{-\infty}^{\infty}\frac{S_{xx}^{2D}(k_x,k_y)\,dk_x\,dk_y}{v^2/(3\kappa_{zz}) + k_\perp^2\kappa_{xx}} \qquad \text{(DD)}. \qquad (19)$$

The analytic theory expressions were evaluated numerically using the MATHEMATICA program (Wolfram Research, Inc.) to perform continuous **k**-space integrals. For the input value of κ_{zz}, we used the simulation value.

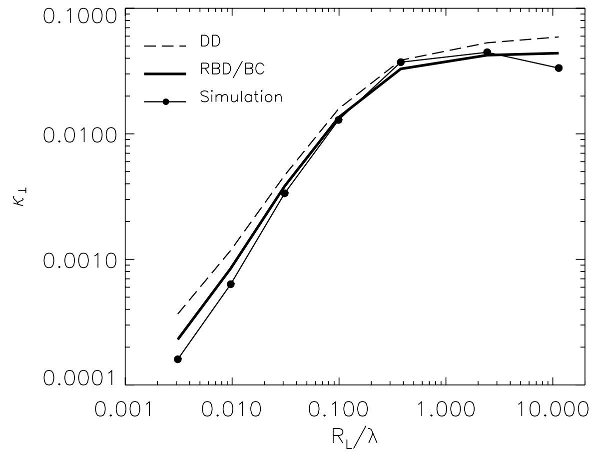

Figure 3. Asymptotic perpendicular diffusion coefficient $\kappa_\perp$ in 2D+slab turbulence with $f_s = 0.2$ and $b/B_0 = 0.5$ as a function of the proton gyroradius in units of the turbulence bendover scale. The simulation values shown here (solid circles) correspond to proton energies ranging from 0.1 MeV to 50 GeV for $B_0 = 5$ nT and $\lambda = 0.02$ AU. In most cases, the RBD/BC theory (thick line) provides a better explanation of the computer simulation results (solid circles) than the original DD theory (dashed line).

5. COMPARISON WITH COMPUTER SIMULATIONS

We have also performed direct computer simulations to trace particle orbits in 2D+slab magnetic turbulence. While the simulations inevitably involve some discretization and statistical errors, they do avoid key assumptions of the analytic work, and thus provide an independent check of their validity.

The computer simulations were performed using the methods, power spectra, and parameter values described by Ruffolo et al. (2008). In particular, all distances are in units of $\lambda = 0.02$ AU, the slab and 2D turbulence bendover scale,[5] and velocities are in units of the speed of light c. Simulations were performed over a sufficient time for all κ_{ii} to approach asymptotic values, within statistical errors. We assume axisymmetry about the large-scale field direction, so κ_{xx} and κ_{yy} should be the same within statistical errors, which we verified in all cases. We report $\kappa_\perp \equiv (\kappa_{xx} + \kappa_{yy})/2$, which can be compared directly with κ_{xx} from theories. In some contexts, we use $\kappa_\perp$ as a synonym for κ_{xx}.

Figure 2 shows the dependence of $\kappa_\perp$ (in units of $c\lambda$) on the overall fluctuation amplitude b/B_0, using $f_s \equiv b_{\rm slab}^2/(b_{\rm slab}^2 + b_{\rm 2D}^2) = 0.2$. It is apparent that the RBD/BC version (thick lines) agrees with computer simulation results (solid circles) better and over a wider range of b/B_0 values than either the DD theory (long-dashed lines) or RBD without the BC (short-dashed lines), over the range of applicability of RBD ($b/B_0 \leqslant 1/a = \sqrt{3}$). We have also examined the dependence on the proton gyroradius (Figure 3), which is related to its energy, for fixed $f_s = 0.2$ and $b/B_0 = 0.5$. The seven simulations were for protons of kinetic energy 0.1, 1, 10, and 100 MeV as well as 1, 10, and 50 GeV. The RBD results, not shown, nearly match DD at $R_L/\lambda < 1$, nearly match RBD/BC at $R_L/\lambda > 1$, and are intermediate at $R_L/\lambda \approx 1$. Overall, the RBD/BC theory again provides the best explanation of the computer simulation results.

[5] Ruffolo et al. (2008) incorrectly specified $\lambda = 0.027$ AU; their simulations actually used $\lambda = 0.02$ AU, and calculations were performed for the same parameters as the simulations.

6. DISCUSSION

In the present work, we interpret NLGC theory in terms of particle guiding center trajectories that are ballistic with constant velocity over the distance scale leading up to their decorrelation (Figure 1), a standard assumption in random walk theory based on scattering concepts. Such RBD stands in contrast to the previous assumption of DD in which the displacements were taken to spread according to asymptotic diffusion. The use of Corrsin's hypothesis for RBD is similar in spirit to a Fokker–Planck approach in which the unperturbed trajectory has a constant but random velocity whose directional distribution is related to the distribution of magnetic fluctuations. It is also related to the Langevin-equation approaches of Balescu et al. (1994). Our use of a heuristic BC that is specific to RBD leads to a substantial improvement in the match with direct computer simulation results, compared with DD and RBD without BC.

Note that RBD theory does not require a small fluctuation amplitude, and indeed RBD/BC matches computer simulation results very well for amplitudes up to $b/B_0 \sim 1$ (Figure 2). The inapplicability for $b/B_0 > 1/a = \sqrt{3}$ indicates room for future improvements to obtain a truly non-perturbative theory. At the same time, we should note that the NLGC framework treats only transverse magnetic fluctuations. In the interplanetary medium of the inner heliosphere, transverse fluctuations account for $\sim$90% of the magnetic fluctuation energy (Belcher & Davis 1971), so NLGC is well justified in this case. However, for large amplitudes with $b/B_0 \gg 1$ there is little reason for the fluctuations to be so strongly anisotropic, and the NLGC framework itself may have limited applicability.

Considering the dependence of $\kappa_\perp$ on the proton Larmor radius, R_L, as shown in Figure 3, a discrepancy remains between NLGC theory and simulation results for the two lowest energies, 0.1 and 1 MeV. The discrepancy is substantially reduced for RBD/BC. For energies of 10 MeV to 10 GeV (i.e., $R_L/\lambda = 0.031$–2.4), RBD/BC theory matches the simulation results very well. The increase with R_L/λ saturates in this range because $\kappa_\perp$ is roughly proportional to v (Minnie et al. 2009), which saturates at c.

The NLGC framework in general could break down when $R_L/\lambda \gg 1$. In this weak scattering limit NLGC considers that guiding center motion tracks the local field line random walk, whereas such a large gyroradius implies that particles experience fluctuations independent from those at the guiding center, and low-wavelength fluctuations should have less influence on perpendicular diffusion when they are averaged over such a large gyroradius. In the interplanetary magnetic field near Earth of about 5 nT with $\lambda \sim 0.02$ AU (Jokipii & Coleman 1968), we have $R_L \sim \lambda$ for a proton energy of about 4 GeV, and in the local galactic magnetic field of about 0.4 nT (Opher et al. 2009), where $\lambda \sim 100$ pc (Armstrong et al. 1995; Dyson & Williams 1997), we have $R_L \sim \lambda$ for a proton energy of $\sim 4 \times 10^{17}$ eV.

We have searched for and found this effect at the highest proton energy considered, 50 GeV, which corresponds to $R_L/\lambda = 11$ for our parameter values of $B_0 = 5$ nT, $b/B_0 = 0.5$, and $f_s = 0.2$, which are applicable to the interplanetary medium near Earth. The perpendicular diffusion coefficient $\kappa_\perp$ decreases, presumably due to cancellation of low-wavelength fluctuations over the gyro-orbit, while all NLGC theories predict a slight increase. In any case, the above energies where $R_L \sim \lambda$ for interplanetary and interstellar propagation are sufficiently high that NLGC theories remain applicable to a wide range of cosmic ray and energetic particle transport problems.

This work was partially supported by the Development and Promotion of Science and Technology Talents Project of the Royal Thai Government, the U.S. NSF (AGS-1063439 and SHINE AGS-1156094), NASA (Heliophysics Theory NNX11AJ4G), the Solar Probe Plus/ISIS project, and the Thailand Research Fund and Thailand's Commission on Higher Education, Ministry of Education (MRG 5286239). We thank Achara Seripienlert for technical assistance.

REFERENCES

Armstrong, J. W., Rickett, B. J., & Spangler, S. R. 1995, ApJ, 443, 209
Axford, W. I. 1965, Planet. Space Sci., 13, 115
Balescu, R., Wang, H.-D., & Misguich, J. H. 1994, Phys. Plasmas, 1, 3826
Belcher, J. W., & Davis, L., Jr. 1971, J. Geophys. Res., 76, 3534
Bieber, J. W., Matthaeus, W. H., Shalchi, A., & Qin, G. 2004, Geophys. Res. Lett., 31, L10805
Bieber, J. W., Matthaeus, W. H., Smith, C. W., et al. 1994, ApJ, 420, 294
Bieber, J. W., Wanner, W., & Matthaeus, W. H. 1996, J. Geophys. Res., 101, 2511
Borovsky, J. E. 2008, J. Geophys. Res., 113, A08110
Corrsin, S. 1959, in Atmospheric Diffusion and Air Pollution, ed. F. Frenkel & P. Sheppard (Advances in Geophysics, Vol. 6; New York: Academic), 161
Dyson, J. E., & Williams, D. A. (ed.) 1997, in The Graduate Series in Astronomy, The Physics of the Interstellar Medium (2nd ed.; Bristol: Institute of Physics Publishing)
Ghilea, M. C., Ruffolo, D., Chuychai, P., et al. 2011, ApJ, 741, 16
Gleeson, L. J. 1969, Planet. Space Sci., 17, 31
Gray, P. C., Pontius, D. H., Jr., & Matthaeus, W. H. 1996, Geophys. Res. Lett., 23, 965
Green, M. S. 1951, J. Chem. Phys., 19, 1036
Jokipii, J. R. 1966, ApJ, 146, 480
Jokipii, J. R., & Coleman, P. J. 1968, J. Geophys. Res., 73, 5495
Kubo, R. 1957, J. Phys. Soc. Japan, 12, 570
le Roux, J. A., & Webb, G. M. 2007, ApJ, 667, 930
Matthaeus, W. H., Goldstein, M. L., & Roberts, D. A. 1990, J. Geophys. Res., 95, 20673
Matthaeus, W. H., Gray, P. C., Pontius, D. H., Jr., & Bieber, J. W. 1995, Phys. Rev. Lett., 75, 2136
Matthaeus, W. H., Qin, G., Bieber, J. W., & Zank, G. 2003, ApJ, 590, L53
Minnie, J., Bieber, J. W., Matthaeus, W. H., & Burger, R. A. 2007, ApJ, 663, 1049
Minnie, J., Matthaeus, W. H., Bieber, J. W., Ruffolo, D., & Burger, R. A. 2009, J. Geophys. Res., 114, A01102
Opher, M., Alouani Bibi, F., Toth, G., et al. 2009, Nature, 462, 1036
Qin, G. 2007, ApJ, 656, 217
Qin, G., Matthaeus, W. H., & Bieber, J. W. 2002a, Geophys. Res. Lett., 29, 7
Qin, G., Matthaeus, W. H., & Bieber, J. W. 2002b, ApJ, 578, L117
Ruffolo, D., Chuychai, P., Wongpan, P., et al. 2008, ApJ, 686, 1231
Salu, Y., & Montgomery, D. C. 1977, Phys. Fluids, 20, 1
Seripienlert, A., Ruffolo, D., Matthaeus, W. H., & Chuychai, P. 2010, ApJ, 711, 980
Shalchi, A. 2006, A&A, 453, L43
Shalchi, A. 2010, ApJ, 720, L127
Shalchi, A., Bieber, J. W., Matthaeus, W. H., & Qin, G. 2004, ApJ, 616, 617
Shalchi, A., Bieber, J. W., Matthaeus, W. H., & Schlickeiser, R. 2006, ApJ, 642, 230
Shebalin, J. V., Matthaeus, W. H., & Montgomery, D. 1983, J. Plasma Phys., 29, 525
Taylor, G. I. 1922, Proc. London Math. Soc. Ser. 2, 20, 196
Weygand, J. M., Matthaeus, W. H., Dasso, S., et al. 2009, J. Geophys. Res., 114, A07213