JOURNAL OF GEOPHYSICAL RESEARCH, VOL. 114, A01102, doi:10.1029/2008JA013349, 2009

When do particles follow field lines?

J. Minnie,[1] W. H. Matthaeus,[1] J. W. Bieber,[1] D. Ruffolo,[2] and R. A. Burger[3]

Received 28 April 2008; revised 2 September 2008; accepted 7 November 2008; published 9 January 2009.

[1] We examine charged particle transport perpendicular to the large scale magnetic field. We find that the limit of an infinite parallel mean free path of particles diffusing along the large scale magnetic field is a necessary condition for which the diffusive spread of the magnetic field lines leads to a proportional spread of the particles. When it occurs this requires that parallel mean free path is well in excess of the smaller of the system size and the turbulence ultrascale. However, there are alternative situations in which particles may diffuse, but field lines do not. In the latter cases the asymptotic behavior is that which persists after the parallel mean free path exceeds some multiple of the correlation scales. This phenomenon of diffusing particles/non-diffusing field lines is typically determined by the 2D turbulence spectrum, where the diffusion coefficient of the magnetic field due to 2D turbulence can diverge if the spectrum of the 2D fluctuations is not well behaved at small wave numbers. We also show that the classical relation between parallel and perpendicular diffusion for high energy particles is consistent with the field line random walk description of particle diffusion.

Citation: Minnie, J., W. H. Matthaeus, J. W. Bieber, D. Ruffolo, and R. A. Burger (2009), When do particles follow field lines?, *J. Geophys. Res.*, *114*, A01102, doi:10.1029/2008JA013349.

1. Introduction

[2] It is often stated that charged particle trajectories follow magnetic field lines in a plasma. There are several possible meanings to this proposition and a number of ways in which they can be violated. For example, the so-called Field Line Random Walk (FLRW) model of particle transport [e.g., *Jokipii*, 1966] is valid when particle gyrocenters follow the meandering field lines at a roughly constant speed. When the field lines spread diffusively, so also will the particles, and this can then be called "first diffusion." Both scattering and drift effects [*Rossi and Olbert*, 1970] lead to violations of this simple picture. Nevertheless the basic idea is often employed or adapted with apparent success, either in the global sense of particles behaving like "beads on a string" and retracing their steps when their motion along the field is reversed by scattering effects (subdiffusion), or in the sense of locally "following field lines" for at least a short distance. On the other hand there are circumstances in which departures from "particles following field lines" are important, and notably there are cases in which such departures are required to obtain "second diffusion" in perpendicular transport [*Rechester and Rosenbluth*, 1978; *Chandran and Cowley*, 1998; *Qin et al.*, 2002a; *Matthaeus et al.*, 2003]. In this paper, we address the question posed in the title by seeking to provide at least partial clarification of the question of whether, and in what way, particles "follow field lines" or behave like "beads on a string" in certain limits of perpendicular diffusion. We shall do this by presenting results from numerical simulations and by revisiting some recent theoretical treatments of particle diffusion.

[1] Bartol Research Institute, Department of Physics and Astronomy, University of Delaware, Newark, Delaware, USA.

[2] Department of Physics, Faculty of Science, Mahidol University, Bangkok, Thailand.

[3] Unit for Space Physics, School of Physics, North-West University, Potchefstroom, South Africa.

0148-0227/09/2008JA013349$09.00

2. Beads on a String?

[3] The elementary notion of particles following field lines emerges in single particle orbit theory [e.g., *Rossi and Olbert*, 1970] where the gyrocenter of charged particle motion remains on a certain magnetic field line when that field is uniform and constant, and the electric field is negligible. When constant (or slowly varying) magnetic field gradients and constant electric fields are present, (gyroperiod-averaged) drift velocities provide a correction to the simplest picture, and in the same limit adiabatic invariants (e.g., magnetic moment) provide useful constraints on possible particle motions. Even in these idealized circumstances, gyrocenter trajectories can become undefined when the field lines themselves become ambiguous, for example when neutral points or separatrices of the magnetic field are present. Moreover when symmetries of the magnetic field are imposed [*Jokipii et al.*, 1993; *Jones et al.*, 1998] the idea that "particles remain on a specific field line" can be replaced by "particles remain on a flux surface", that is, somewhere on a set of equivalent field lines. The situation becomes more complicated when classical (hard sphere or Coulomb) scattering is introduced, and conditions for the drifting gyrocenter picture can be strongly violated.

[4] It is in the midst of this already complex landscape that one seeks to develop transport theories for charged particles in a low-collisionality turbulent medium. Quasi-linear (QLT) or Fokker-Planck approaches [*Jokipii*, 1966; *Schlickeiser*, 1989] develop perturbation schemes in which random forces on particles are computed along unperturbed trajectories. Here one begins to see how problems interpreting the beads-on-a-string picture arise: pitch angle scattering changes the particle magnetic moment, and also, in fully three dimensional cases, changes the bundle of field lines encircled by the gyro-orbit. Therefore there is a change in the range of possibilities of "which field line to follow" when even weak scattering is present.

[5] For stronger scattering [*Lingenfelter et al.*, 1971; *Urch*, 1977; *Rechester and Rosenbluth*, 1978] the parallel and perpendicular scattering processes are no longer independent. Scattering parallel to the mean magnetic field can cause a reversal of the particle velocity along the magnetic field, a subsequent reduction of perpendicular random displacement, and the possibility of subdiffusion. This process is observed in simulations [*Qin et al.*, 2002b]. Also seen in some cases is the restoration of diffusive transport [*Qin et al.*, 2002a] when the particles effectively change which field line they are following, and the magnetic turbulence exhibits sufficient spatial complexity. In such cases a diffusive limit can also be established, once the displacements become uncorrelated, although the reasons for the decorrelation are different from the case of particles simply following field lines. This phenomenon of second diffusion, which replaces perpendicular subdiffusion at longer time intervals for spatially complex turbulence, is reasonably well described by the Nonlinear Guiding Center Theory [NLGC; *Matthaeus et al.*, 2003; *Bieber et al.*, 2004] and its offspring [e.g., *Shalchi et al.*, 2004b]. In this picture one can say that the particle guiding centers locally follow field lines over a distance determined by the mean free paths of parallel scattering and the field line random walk. After that, the particle switches to a different field line. The guiding center motion is taken to be randomized, with no "backtracking" along the same field line.

3. FLRW Limit

[6] For reasons that will presently become clear, in what follows we define the field line random walk (FLRW) limit of particle diffusion to be all cases in which the particle perpendicular diffusion coefficient $\kappa_\perp$ is related to the magnetic field line diffusion coefficient (or Fokker-Planck coefficient) $D_\perp$ through the proportionality

$$\kappa_\perp \propto vD_\perp, \tag{1}$$

with v the particle speed. One can arrive readily at this form by a simple heuristic argument: Suppose that on average the magnetic field spreads a mean square perpendicular distance $\langle(\Delta x)^2\rangle$ per unit distance Δz along the mean magnetic field. Further suppose that particle gyrocenters follow field lines, so that the particle perpendicular diffusion coefficient is estimated as

$$\kappa_\perp = \left\langle \left| \frac{\Delta z_p}{\Delta t} \right| \right\rangle \frac{\left\langle (\Delta x)^2 \right\rangle}{2\Delta z}. \tag{2}$$

Here z_p denotes particle position along the mean field. The effective speed of movement v_{eff} along z needs to be determined by further considerations, but in any case can be scaled to the total particle speed, so that $v_{eff} = \langle|\Delta z_p/\Delta t|\rangle = qv$ for some number q. Then, when the Fokker-Planck coefficient for field line random walk $D_\perp = \langle(\Delta x)^2\rangle/(2\Delta z)$ is well defined, we arrive at particle transport as in equation (1). Here we consider the term FLRW transport, defined by equation (1), to describe a class of perpendicular transport models that can in principle have different constants of proportionality, q, in different circumstances.

[7] For "first diffusion" of particles traveling over time intervals less than a scattering mean free time, we expected that $\kappa_\perp = \langle|\mu|\rangle vD_\perp$, where μ is the cosine of the pitch angle between the particle momentum and the average magnetic field. For example, a beam of solar energetic particles moving at low pitch angles along the magnetic field over a distance less than a mean free path could have $|\mu|$ close to 1, and after the same population is isotropized by scattering, then subsequent perpendicular diffusion over times less than a scattering mean free time would be governed by $\langle|\mu|\rangle = 1/2$. In such cases the particle perpendicular mean free path is on the order of the field line diffusion coefficient: $\lambda_\perp \sim \kappa_\perp/v \sim D_\perp$. With strong parallel scattering, one obtains subdiffusive perpendicular transport [e.g. *Urch*, 1977; *Kóta and Jokipii*, 2000; *Qin et al.*, 2002b], or levels of particle perpendicular diffusion in which particles spread more slowly than in the FLRW limit with $\langle|\mu|\rangle = 1/2$ [e.g. *Giacalone and Jokipii*, 1999; *Qin et al.*, 2002a; *Matthaeus et al.*, 2003; *Minnie*, 2006]. Various factors can contribute to establishing the effective speed of particles along field lines and the constant of proportionality in equation (1), the most obvious of which are related to the steady three dimensional particle velocity distribution that is obtained in a particular physical situation or numerical experiment. For example, the *Jokipii* [1966; QLT] result corresponds to $\kappa_\perp = vD_\perp/2$, that is, an effective speed of $v/2$ as discussed above. Throughout the paper we maintain the conventional relation between diffusion coefficient and mean free path, namely $\kappa_\perp = v\lambda_\perp/3$.

[8] In the following sections, we will compare the transport of particles and field lines to examine when a FLRW limit is obtained, that is, when the diffusive spread of particles across the large-scale field is proportional to the diffusive spread of field lines. This requires that we examine cases with varying parallel mean free paths. We find, perhaps surprisingly, that only in some cases is FLRW the asymptotic behavior, and in others particle transport may be diffusive even when magnetic field line random walk is not diffusive. This leads to the conclusion that while particle gyrocenter trajectories have a strong relation to field line trajectories, there are other factors that can prevent particles and field lines from diffusing at proportionally similar rates.

4. Numerical Experiments

[9] The results presented here are from simulations by *Minnie* [2006], also discussed in *Minnie et al.* [2007]. The simulations cover magnetic fluctuation amplitudes δB ranging from 0.2 to 5.0, in units of the background magnetic field magnitude B_0 and a range of maximal (90° pitch-angle) particle Larmor radii r_M from 0.01 to 1.0, in units of

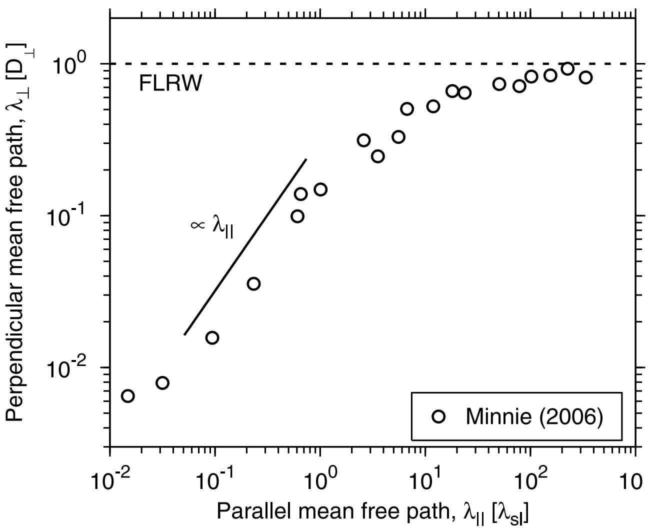

Figure 1. Simulation results showing the perpendicular mean free path as a function of the parallel mean free path. Here the perpendicular mean free path has been normalized to the diffusion coefficient of the magnetic field lines ($D_\perp$) and the parallel mean free path has been normalized to the bend-over scale of the slab turbulence power spectrum (λ_{sl}).

the bend-over scale λ_{sl} of the slab turbulence power spectrum. The bend-over scale the length-scale λ associated with the change of shape of the spectrum at the low wavenumber end of the inertial range. The latter is frequently modeled as a power law. That is, for $k\lambda > 1$, with k the wave number, the spectrum is of the inertial range form. The bend-over is typically, but not always, comparable to the correlation scale of the fluctuations [*Matthaeus et al.*, 2007]. The turbulence is a composite of slab and two-dimensional (2D) geometry [e.g. *Gray et al.*, 1996], with 20% of the fluctuation energy in the slab component and 80% in the 2D component, typical of near-Earth solar wind conditions [*Bieber et al.*, 1996]. In the simulations the bend-over scale of the slab turbulence power spectrum was used as the unit of length. The value of the bend-over scale of the 2D turbulence power spectrum, λ_{2D}, was chosen to be $\lambda_{2D} = 0.1\lambda_{sl}$.

[10] In Figure 1 we show the particle perpendicular mean free path normalized to the value of the field line diffusion coefficient, as a function of the particle parallel mean free path (which is in turn dependent upon δB). The horizontal dashed line indicates $\lambda_\perp = D_\perp$, corresponding to a FLRW model with $\kappa_\perp = (1/3)vD_\perp$, and an effective speed equal to a third of the total particle speed.

[11] Note that in what follows, the value for the field line diffusion coefficient is not determined directly from the simulations. Instead, the theoretical value of $D_\perp$ from *Matthaeus et al.* [1995] is used throughout the present discussion, using the parameters of the model magnetic field of the simulations. The theoretical predictions were shown to closely correspond to computer simulation results [*Gray et al.*, 1996]. This issue will be addressed in more detail in a subsequent section.

[12] In Figure 1, there is a very clear trend of $\lambda_\perp$ approaching $D_\perp$ as $\lambda_{||}$ becomes sufficiently large. A sufficient condition for this would be that particles follow field lines as "beads on a string;" however, this condition is not necessary. In fact the current view is that particles traveling longer than a mean free time must deviate from following individual field lines in order to avoid subdiffusion [*Urch*, 1977; *Rechester and Rosenbluth*, 1978] and to obtain second diffusion [*Qin et al.*, 2002a]. Therefore at this point we can only assert that the limiting behavior seen in Figure 1 implies that the statistical spread of particles perpendicular to the mean magnetic field becomes similar to the transverse spread of field lines.

5. Energy Effects on "Beads"

[13] Intuitively one might expect that a particle with lower energy (smaller Larmor radius) will follow the field more closely than its high energy (larger Larmor radius) counterpart. Indeed, the derivation of gyrocenter and drift equations of motion starting from smooth fields supports this predisposition. However, this is not always true.

[14] On the one hand, arbitrarily low energy electrons can exhibit an increasing parallel mean free path with decreasing energy, associated with their resonant interaction with the steepened power spectrum in the dissipation range of the turbulence [*Bieber et al.*, 1994; *Dröge*, 2000]. When parallel scattering is suppressed in this way, the electrons can follow a single field line for a longer time, leading toward a tendency of $\lambda_\perp \propto D_\perp$.

[15] However, it is also significant that the parallel mean free path of electrons and protons at arbitrarily high energy is typically a monotonically increasing function of energy. For particles resonant with the turbulence inertial range, this is readily deduced from theoretical treatments [e.g. *Jokipii*, 1971; *Bieber et al.*, 1995; *Shalchi et al.*, 2004b] and numerical simulations [e.g. *Giacalone and Jokipii*, 1999; *Qin*, 2002; *Minnie et al.*, 2007] of particle transport in turbulent magnetic fields. Consequently, it becomes apparent that the limit of large $\lambda_{||}$ in Figure 1 corresponds to *a high energy* approach to a FLRW transport limit. Somewhat paradoxically, one sees that the notion of particles "following field lines," in this statistical sense, need not necessarily be a small Larmor radius effect, but rather, can be a high energy, large Larmor radius limit.

6. Analytical Examination of the FLRW Limit

[16] To understand Figure 1 and the approach of $\lambda_\perp$ to a stable value of order $D_\perp$ as $\lambda_{||}$ becomes large, we turn to the nonlinear guiding center (NLGC) theory of asymptotic (second) diffusion [*Matthaeus et al.*, 2003]. The NLGC theory has been used successfully in an ab initio cosmic ray modulation model to account for the rigidity dependence of the observed latitudinal gradients of protons in the heliosphere [*Minnie et al.*, 2005]. It has also been successful in accounting for the inferred perpendicular mean free path of Jovian electrons arriving at Earth [*Bieber et al.*, 2004].

[17] For the case of axisymmetric perpendicular diffusion (i.e. $\kappa_{xx} = \kappa_{yy} \equiv \kappa_\perp$) and a static magnetic field, the NLGC perpendicular diffusion coefficient satisfies

$$\kappa_\perp = \frac{a^2 v^2}{6B_0^2} \int_{-\infty}^{\infty} \mathrm{d}^3k \, \frac{S(\mathbf{k})}{v/\lambda_{||} + k_z^2\kappa_{||} + k_\perp^2\kappa_\perp}, \tag{3}$$

with a a numerical constant of order unity, v the particle speed, $\mathbf{B}_0 = B_0\hat{\mathbf{z}}$ the large scale magnetic field and $S(\mathbf{k}) \equiv S_{xx}(\mathbf{k}) + S_{yy}(\mathbf{k})$ the total power spectrum of the magnetic fluctuations, i.e., the sum of spectra of fluctuations in the x- and y-directions, as a function of the wave vector $\mathbf{k} = (k_x, k_y, k_z)$. Also, we denote by $k_\perp$ the perpendicular wavenumber with $k_\perp^2 = k_x^2 + k_y^2$.

[18] Using the composite model for the magnetic fluctuations, the fluctuation spectrum can be written as $S(\mathbf{k}) = S_{sl}(\mathbf{k})\delta(k_x)\delta(k_y) + S_{2D}(\mathbf{k})\delta(k_z)$. Using the relation $\lambda_\perp = 3\kappa_\perp/v$, we can therefore write equation (3) as

$$\lambda_\perp = \frac{a^2}{2B_0^2}\left[\int_{-\infty}^{\infty} \frac{\mathrm{d}k_z S_{sl}(k_z)}{1/\lambda_\parallel + k_z^2\lambda_\parallel/3} + \int_0^{\infty} \frac{\mathrm{d}k_\perp\, 2\pi k_\perp S_{2D}(k_\perp)}{1/\lambda_\parallel + k_\perp^2\lambda_\perp/3}\right]. \quad (4)$$

[19] One important property of the NLGC formulation is that it includes the influence of $\lambda_\parallel$ on the determination of $\lambda_\perp$, thus incorporating at least some of the essential physics of the interplay between these types of transport, the nature of which we alluded to in the introductory paragraphs above. We are particularly interested in the behavior of $\lambda_\perp$ when parallel scattering becomes negligible, i.e., $\lambda_\parallel \to \infty$. To examine this, we normalize all length scales to a characteristic length scale L_0, which is assumed to be independent of $\lambda_\parallel$, define the normalized wave numbers $k_z^\star = k_z L_0$ and $k_\perp^\star = k_\perp L_0$, and introduce $\epsilon = L_0/\lambda_\parallel$. The spectra are normalized as $S_{sl}^\star(k_z^\star) = S_{sl}(k_z)/L_0$ and $S_{2D}^\star(k_\perp^\star) = S_{2D}(k_\perp)/L_0^2$.

[20] The total normalized perpendicular mean free path $\lambda_\perp^\star \equiv \lambda_\perp/L_0$ in equation (4) is therefore of the form $\lambda_\perp^\star = I^{sl}(\epsilon) + I^{2D}(\epsilon, \lambda_\perp^\star)$, with I^{sl} and I^{2D} depending on the fluctuation spectra, and understood to be normalized to the characteristic length scale L_0.

[21] In the limit $\epsilon \to 0$ the slab term contribution to $\lambda_\perp^\star$ in equation (4), $I_0^{sl} \equiv \lim_{\epsilon\to 0} I^{sl}(\epsilon)$, becomes

$$I_0^{sl} = \frac{a^2}{2B_0^2}\lim_{\epsilon\to 0}\int_{-\infty}^{\infty} \frac{\mathrm{d}k_z^\star\, S_{sl}^\star(k_z^\star)\epsilon}{\epsilon^2 + k_z^{\star 2}/3} = \frac{\sqrt{3}\pi a^2}{2B_0^2}S_{sl}^\star(0), \quad (5)$$

since $\lim_{\epsilon\to 0}\epsilon/(\epsilon^2 + q^2) = \pi\delta(q)$, the Dirac delta function. The diffusion coefficient of magnetic field lines due to slab turbulence is given by $D_{sl}^\star = \pi S_{sl}^\star(0)/(2B_0^2)$ [e.g. *Jokipii*, 1966], and thus here $I_0^{sl} = \sqrt{3}a^2 D_{sl}^\star$.

[22] The 2D contribution to $\lambda_\perp^\star$ in equation (4) is

$$I^{2D}(\epsilon, \lambda_\perp^\star) = \frac{a^2}{2B_0^2}\int_0^{\infty} \mathrm{d}k_\perp^\star\, \frac{2\pi k_\perp^\star S_{2D}^\star(k_\perp^\star)}{\epsilon + \lambda_\perp^\star k_\perp^{\star 2}/3}. \quad (6)$$

[23] The limit of equation (6) when $\epsilon \to 0$ may or may not converge, depending on the behavior of $S_{2D}^\star(k_\perp^\star)$ at small wave numbers.

[24] In this regard, it is notable that a number of studies have incorporated 2D and quasi-2D spectrum models that are finite when evaluated in a periodic box, but become singular (but with finite energy) in the above sense when passing to the unbounded limit [see also, *Matthaeus et al.*, 2007].

[25] Suppose the normalized modal 2D spectrum behaves as $S_{2D}^\star(k_\perp^\star) \sim k_{\perp p}^\star$ as $k_\perp^\star \to 0$. Then, if $p > 0$ the integral $I^{2D}(\epsilon \to 0, \lambda_\perp^\star) \equiv I^{2D}{}_0(\lambda_\perp^\star)$ is well behaved and one obtains

$$I_0^{2D}(\lambda_\perp^\star) = \frac{3a^2}{\lambda_\perp^\star}\left[\frac{1}{2B_0^2}\int_0^{\infty}\mathrm{d}k_\perp^\star\, \frac{2\pi S_{2D}^\star(k_\perp^\star)}{k_\perp^\star}\right]. \quad (7)$$

The quantity in square brackets is $D_{2D}^{\star 2}$, the square of the diffusion coefficient of the magnetic field lines due to 2D turbulence [*Matthaeus et al.*, 1995; *Ruffolo et al.*, 2004], implying $I_0^{2D}(\lambda_\perp^\star) \sim D_{2D}^{\star 2}/\lambda_\perp^\star$.

[26] Another possibility is that $p \leq 0$, in which case the diffusion coefficient $D_{2D}^\star$ of the field lines is divergent, as is the $\epsilon \to 0$ limit of equation (6). In this case one can impose a lower limit on $k_\perp^\star$ given by L_0/L, where L is a measure of the system size which provides an effective cutoff. The energy remains finite as long as $p > -2$. Then the 2D integral expression in equation (7) scales as $\sim\log(L/L_0)$ when $p = 0$, and as $\sim(L/L_0)^{|p|}$ for $p < 0$. In these cases both the field line diffusion and the contribution to particle diffusion from 2D turbulence are determined by the system size L. Nevertheless, with the cutoff procedure in place, the idea of field line diffusion is restored and the 2D contribution may still be written as a finite (but perhaps very large) quantity $D_{2D}^\star$. A similar cutoff procedure might be introduced for the slab contribution (equation (5)) when the integral diverges, because the quantity $S_{sl}^\star(0) \propto \lambda_{sc}$ is unbounded, with λ_{sc} the correlation length of the turbulence. However we would expect the slab fluctuations to have finite energy and become uncorrelated at large separations and this leads to a finite slab contribution.

[27] Consequently, in either the well behaved case, or divergent cases with a large scale cutoff, the two separate limits (i.e. $I_0^{sl} + I_0^{2D}(\lambda_\perp^\star)$) combine to yield

$$\lambda_\perp^\star = \sqrt{3}a^2 D_{sl}^\star + 3a^2 D_{2D}^{\star 2}/\lambda_\perp^\star. \quad (8)$$

This equation is nearly identical to the quadratic equation for the total diffusion coefficient of magnetic field lines obtained by *Matthaeus et al.* [1995], except that here it refers to a limiting form of NLGC particle diffusion, namely, the limit when the parallel mean free path tends to infinity.

[28] If the 2D turbulence is dominant, as it is in the solar wind, the second term on the r.h.s. of equation (8) is dominant and in this case NLGC predicts an asymptotic high energy (weak scattering) behavior that is very close to an FLRW limit. Furthermore, if one assumes that $a^2 = 1/3$, NLGC yields exactly the special case of FLRW transport discussed above, namely $\lambda_\perp = D_{2D}$. It is of interest to note that this value of $a^2 = 1/3$ has been found empirically from numerical simulations to adequately describe perpendicular particle diffusion, especially in strong turbulence when the particles are frequently backscattered along the magnetic field [*Matthaeus et al.*, 2003; *Minnie et al.*, 2007]. Thus, we

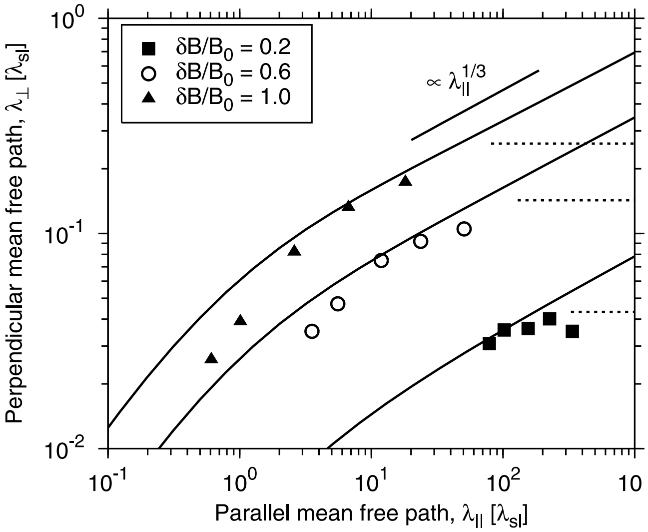

Figure 2. Comparison between simulation results (symbols) and theoretical predictions from the nonlinear guiding center theory of *Matthaeus et al.* [2003] (solid lines). The dashed lines are the values for the field line diffusion coefficients from the simulations. The symbols indicate that the perpendicular mean free path is saturating at a value determined by the box size L, rather than behaving as an increasing power-law in $\lambda_\parallel$ as it presumably would in an infinite domain with the assumed spectral law extending to infinitely large scale. Note that the diffusion coefficient for the field lines would also diverge in this case, as the ultrascale would diverge as $\sqrt{L}$ as box size $L \to \infty$.

expect that the limiting form of perpendicular particle diffusion at large parallel mean free path will be close to an FLRW-type transport with $\kappa_\perp \propto vD_\perp$.

7. Classical Scattering

[29] As an aside, we discuss the classical scattering relation [e.g. *Gleeson*, 1969]

$$\lambda_\perp = \frac{\lambda_\parallel}{1 + \left(\lambda_\parallel / r_M\right)^2}, \tag{9}$$

where r_M is the maximal Larmor radius, i.e., 90° pitch-angle particles. We shall look especially at the applicability of this theory to the description of perpendicular diffusion at high particle rigidities. *Giacalone and Jokipii* [1999] concluded that the classical scattering relation above is the appropriate value to use for the perpendicular mean free path for high rigidity particles.

[30] From quasilinear theory (QLT) [e.g. *Jokipii*, 1966; *Zank et al.*, 1998; *Giacalone and Jokipii*, 1999] and the weakly nonlinear theory (WNLT) [*Shalchi et al.*, 2004b] the parallel mean free path is expected to scale as r_M^2 for high rigidity particles. This implies that the ratio $\lambda_\parallel / r_M$ will increase with increasing particle rigidity, which results in

$$\lambda_\perp \simeq \frac{r_M^2}{\lambda_\parallel}, \tag{10}$$

at high rigidities. From all of the QLT references above one finds a parallel mean free path on the order of

$$\lambda_\parallel \sim \frac{r_M^2}{\lambda_c} \frac{B_0^2}{\delta B^2}, \tag{11}$$

with λ_c and δB^2 the correlation scale and variance of the turbulence, respectively, and B_0 the magnitude of the background magnetic field. Using this value for the parallel mean free path in equation (10) one immediately obtains

$$\lambda_\perp \sim \lambda_c \frac{\delta B^2}{B_0^2}, \tag{12}$$

which is consistent with the FLRW limit of particle diffusion.

[31] It is therefore our contention that the classical scattering relation between parallel and perpendicular diffusion coefficients is appropriate at sufficiently large rigidities *because* the parallel mean free path is sufficiently large *and* the particles are tending to follow the field lines.

8. Analytic Solutions and Simulation Results in a Finite Domain

[32] We have seen above that the NLGC theory of perpendicular transport, embodied by equation (3), admits the possibility that FLRW transport is recovered when $\lambda_\parallel \to \infty$, usually in the high energy limit. However, previous analytical solutions of equation (3) lead to $\lambda_\perp \propto \lambda_\parallel^{1/3}$ when $\lambda_\parallel \to \infty$ [*Shalchi et al.*, 2004a; *Zank et al.*, 2004], which of course implies that $\lambda_\perp \to \infty$. The result in equation (8) therefore seems to be contradictory to the results from previous studies.

[33] Based on the above discussion, we can see that this apparent discrepancy is easily resolved by examination of the large scale behavior of the fluctuation spectra. The resolution is given most effectively by example. The behavior $\lambda_\perp \propto \lambda_\parallel^{1/3}$ at large $\lambda_\parallel$ results from equation (6) when using a spectrum with $p = -1$. In Figure 2 we present solutions of equation (4) for different values of the magnetic fluctuation amplitude as a function of the parallel mean free path. The solutions of equation (4), denoted by the solid lines, clearly increase monotonically with increasing $\lambda_\parallel$, becoming larger than the appropriate FLRW limits denoted by the dashed lines at some large value of $\lambda_\parallel$. The symbols are the simulation results from *Minnie* [2006]. One can see that the numerical results, necessarily computed in a finite size box, roll over to the flat behavior expected of FLRW-type perpendicular transport. This occurs even though the functional form of the 2D spectrum is similar to that used by *Shalchi et al.* [2004a] and *Zank et al.* [2004] except that in this case the spectral density includes a minimum wavenumber $1/L$, where L is the box size. Consequently, the infinite homogeneous limit in which $\lambda_\perp \propto \lambda_\parallel^{1/3}$ is obtained correctly in the analytical theory, but it is not attained in the simulation. Instead, since the value of the spectral index is $p = -1$, the effective value of the ultrascale becomes a quantity of the order of $\sqrt{L}$, and when $\lambda_\parallel \gg L$ by a sufficient margin, FLRW transport is recovered. Consequently, finite size system effects are seen to restore the

"beads on a string" intuitive picture, but perhaps for somewhat more subtle reasons than previously anticipated. In cases in which FLRW perpendicular transport is recovered, we see that this behavior is approached asymptotically as the parallel mean free path exceeds the smaller of the system size or the ultrascale.

9. Discussion and Conclusions

[34] We can now confront, at least partially, what we mean by particles following field lines, and the "FLRW limit". Very low energy particles may sample only very weak turbulence effects and might therefore follow field lines like beads on a string. This is more likely for electrons than protons because their $\lambda_{\|}$ may be very large at low energy due to dissipation range steepening of the fluctuation spectrum [*Bieber et al.*, 1994]. However, when parallel scattering is strong enough, this view must change, and we know from previous work that for a wide range in energy, particles can depart from a random walk proportional to the field line random walk, due to parallel scattering. This can cause subdiffusion, or when the fluctuations have sufficient transverse complexity, diffusion might be restored, but not at a rate proportional to FLRW.

[35] In this paper we have examined several alternatives for perpendicular transport at high energies, based on the behavior of the 2D modal spectrum at very low wavenumber $S(\mathbf{k}_\perp) \sim k_\perp^p$. When the ultrascale is well defined [*Matthaeus et al.*, 2007], for a spectral index $p > 0$, one anticipates that particles of sufficiently large $\lambda_{\|}$ will allow recovery of FLRW transport. If the spectral form is pathological at large scales ($p = -1$) and the ultrascale diverges, the field lines themselves never arrive at a diffusive limit, while particles continue to diffuse, but with $\lambda_\perp \propto \lambda_{\|}^{1/3}$ (in NLGC theory). This is not an FLRW transport regime. However, for $p \leq 0$, a finite system size might introduce a maximum scale that can be sampled, and then this scale fixes an effective ultrascale. In this case the field lines are diffusive, and particle transport approaches an FLRW regime at high energy when the parallel mean free path greatly exceeds the smaller of the ultrascale and the system size. This provides some answers to the question "When do particle follow field lines?" The answer appears to be more involved than might have been anticipated, but this seems to be unavoidable. The transport properties of particles and field lines are intertwined with one another, and the intuition that particles always follow field lines needs to be carefully examined in various regimes of turbulence length scales, system size, and particle energy.

[36] **Acknowledgments.** This research supported in part by NASA grant NNX07AH73G and the Heliophysics Theory Program NNX08AI47G, NSF grants ATM-0539995 and ATM-0752135 (SHINE), and the Thailand Research Fund. RAB acknowledges support by the South African National Research Foundation.

[37] Zuyin Pu thanks Ming Zhang and another reviewer for their assistance in evaluating this paper.

References

Bieber, J. W., W. H. Matthaeus, C. W. Smith, W. Wanner, M.-B. Kallenrode, and G. Wibberenz (1994), Proton and electron mean free paths: The Palmer consensus revisited, *Astrophys. J.*, *420*(1), 294–306.

Bieber, J. W., R. A. Burger, and W. H. Matthaeus (1995), The diffusion tensor throughout the heliosphere, in *Proc. Int. Conf. Cosmic Ray 24th*, IUPAP, Rome, Italy, vol. 4, pp. 694–697.

Bieber, J. W., W. Wanner, and W. H. Matthaeus (1996), Dominant two-dimensional solar wind turbulence with implications for cosmic ray transport, *J. Geophys. Res.*, *101*(A2), 2511–2522.

Bieber, J. W., W. H. Matthaeus, A. Shalchi, and G. Qin (2004), Nonlinear guiding center theory of perpendicular diffusion: General properties and comparison with observation, *Geophys. Res. Lett.*, *31*, L10805, doi:10.1029/2004GL020007.

Chandran, B. D. G., and S. C. Cowley (1998), Thermal conduction in a tangled magnetic field, *Phys. Rev. Lett.*, *80*(14), 3077–3080.

Dröge, W. (2000), The rigidity dependence of solar particle scattering mean free paths, *Astrophys. J.*, *537*(2), 1073–1079.

Giacalone, J., and J. R. Jokipii (1999), The transport of cosmic rays across a turbulent magnetic field, *Astrophys. J.*, *520*(1), 204–214.

Gleeson, L. J. (1969), The equations describing the cosmic-ray gas in the interplanetary region, *Planet. Space Sci.*, *17*, 31–47.

Gray, P. C., D. H. Pontius Jr., and W. H. Matthaeus (1996), Scaling of field-line random walk in model solar wind fluctuations, *Geophys. Res. Lett.*, *23*(9), 965–968.

Jokipii, J. R. (1966), Cosmic-ray propagation: I. Charged particles in a random magnetic field, *Astrophys. J.*, *146*(2), 480–487.

Jokipii, J. R. (1971), Propagation of cosmic rays in the solar wind, *Rev. Geophys. Space Phys.*, *9*(1), 27–87.

Jokipii, J. R., J. Kóta, and J. Giacalone (1993), Perpendicular transport in 1- and 2-dimensional shock simulations, *Geophys. Res. Lett.*, *20*(17), 1759–1761.

Jones, F. C., J. R. Jokipii, and M. G. Baring (1998), Charged-particle motion in electromagnetic fields having at least one ignorable spatial coordinate, *Astrophys. J.*, *509*(1), 238–243.

Kóta, J., and J. R. Jokipii (2000), Velocity correlation and the spatial diffusion coefficients of cosmic rays: Compound diffusion, *Astrophys. J.*, *531*(2), 1067–1070.

Lingenfelter, R. E., R. Ramaty, and L. A. Fisk (1971), Compound diffusion of cosmic rays, *Astrophys. Lett.*, *8*, 93–97.

Matthaeus, W. H., P. C. Gray, D. H. Pontius Jr., and J. W. Bieber (1995), Spatial structure and field-line diffusion in transverse magnetic turbulence, *Phys. Rev. Lett.*, *75*(11), 2136–2139.

Matthaeus, W. H., G. Qin, J. W. Bieber, and G. P. Zank (2003), Nonlinear collisionless perpendicular diffusion of charged particles, *Astrophys. J.*, *590*(1), L53–L56.

Matthaeus, W. H., J. W. Bieber, D. Ruffolo, P. Chuychai, and J. Minnie (2007), Spectral properties and length scales of two-dimensional magnetic field models, *Astrophys. J.*, *67*, 956.

Minnie, J. (2006), An ab initio approach to the heliospheric modulation of galactic cosmic rays, Ph.D. thesis, North-West University (Potchefstroom Campus), South Africa.

Minnie, J., R. A. Burger, S. Parhi, W. H. Matthaeus, and J. W. Bieber (2005), Comparison of the effects of two models for perpendicular diffusion on cosmic-ray latitudinal gradients, *Adv. Space Res.*, *35*, 543–546, doi:10.1016/j.asr.2004.12.001.

Minnie, J., J. W. Bieber, W. H. Matthaeus, and R. A. Burger (2007), On the ability of different diffusion theories to account for directly simulated diffusion coefficients, *Astrophys. J.*, *663*, 1049–1054.

Qin, G. (2002), Charged particle transport in magnetic field turbulence and study of TRIM simulation and SSX experiment, Ph.D. thesis, University of Delaware, Newark, Del.

Qin, G., W. H. Matthaeus, and J. W. Bieber (2002a), Perpendicular transport of charged particles in composite model turbulence: Recovery of diffusion, *Astrophys. J.*, *578*(2), L117–L120.

Qin, G., W. H. Matthaeus, and J. W. Bieber (2002b), Subdiffusive transport of charged particles perpendicular to the large scale magnetic field, *Geophys. Res. Lett.*, *29*(4), 1048, doi:10.1029/2001GL014035.

Rechester, A. B., and M. N. Rosenbluth (1978), Electron heat transport in a Tokamak with destroyed magnetic surfaces, *Phys. Rev. Lett.*, *40*(1), 38–41.

Rossi, B., and S. Olbert (1970), *Introduction to the Physics of Space*, McGraw-Hill, New York.

Ruffolo, D., W. H. Matthaeus, and P. Chuychai (2004), Separation of magnetic field lines in two-component turbulence, *Astrophys. J.*, *614*(1), 420–434.

Schlickeiser, R. (1989), Cosmic-ray transport and acceleration: I. Derivation of the kinetic equation and application to cosmic rays in static cold media, *Astrophys. J.*, *336*(1), 243–263.

Shalchi, A., J. W. Bieber, and W. H. Matthaeus (2004a), Analytic forms of the perpendicular diffusion coefficient in magnetostatic turbulence, *Astrophys. J.*, *604*(2), 675–686.

Shalchi, A., J. W. Bieber, W. H. Matthaeus, and G. Qin (2004b), Nonlinear parallel and perpendicular diffusion of charged cosmic rays in weak turbulence, *Astrophys. J.*, *616*(1), 617–629.

Urch, I. H. (1977), Charged particle transport in turbulent magnetic fields: The perpendicular diffusion coefficient, *Astrophys. Space Sci.*, *46*, 389–406.

Zank, G. P., W. H. Matthaeus, J. W. Bieber, and H. Moraal (1998), The radial and latitudinal dependence of the cosmic ray diffusion tensor in the heliosphere, *J. Geophys. Res.*, *103*(A2), 2085–2097.

Zank, G. P., G. Li, V. Florinski, W. H. Matthaeus, G. M. Webb, and J. A. le Roux (2004), Perpendicular diffusion coefficient for charged particles of arbitrary energy, *J. Geophys. Res.*, *109*, A04107, doi:10.1029/2003JA010301.

J. W. Bieber, W. H. Matthaeus, and J. Minnie, Bartol Research Institute, Department of Physics and Astronomy, University of Delaware, 217 Sharp Laboratory, Newark, DE 19716, USA. (whm@udel.edu; jacominnie@yahoo.com)

R. A. Burger, Unit for Space Physics, School of Physics, North-West University, Private Bag X6001, Potchefstroom 2520, South Africa.

D. Ruffolo, Department of Physics, P603, Faculty of Science, Mahidol University, Rama 6 Rd., Payathai Rajathewee, Bangkok 10400, Thailand.